

Holographic $\Lambda(t)$ CDM model in a non-flat universe

Jing-Fei Zhang,¹ Yang-Yang Li,¹ Ying Liu,¹ Sheng Zou,¹ and Xin Zhang^{*1,2,†}

¹Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China

²Center for High Energy Physics, Peking University, Beijing 100080, China

The holographic $\Lambda(t)$ CDM model in a non-flat universe is studied in this paper. In this model, to keep the form of the stress-energy of the vacuum required by general covariance, the holographic vacuum is enforced to exchange energy with dark matter. It is demonstrated that for the holographic model the best choice for the IR cutoff of the effective quantum field theory is the event horizon size of the universe. We derive the evolution equations of the holographic $\Lambda(t)$ CDM model in a non-flat universe. We constrain the model by using the current observational data, including the 557 Union2 type Ia supernovae data, the cosmic microwave background anisotropy data from the 7-yr WMAP, and the baryon acoustic oscillation data from the SDSS. Our fit results show that the holographic $\Lambda(t)$ CDM model tends to favor a spatially closed universe (the best-fit value of Ω_{k0} is -0.042), and the 95% confidence level range for the spatial curvature is $-0.101 < \Omega_{k0} < 0.040$. We show that the interaction between the holographic vacuum and dark matter induces an energy flow of which the direction is first from vacuum to dark matter and then from dark matter to vacuum. Thus, the holographic $\Lambda(t)$ CDM model is just a time-varying vacuum energy scenario in which the interaction between vacuum and dark matter changes sign during the expansion of the universe.

PACS numbers: 95.36.+x, 98.80.Es, 98.80.-k

I. INTRODUCTION

Dark energy has been one of the most important themes in modern physics. However, today, we are still far from thoroughly understanding the nature of dark energy [1]. The cosmological constant Λ , posited in 1917 and later rejected by Einstein [2], is an important candidate for dark energy, because it can provide a nice explanation for the accelerating universe and can fit the observational data well [3]. The cosmological model containing the cosmological constant Λ and cold dark matter (CDM) is known as the Λ CDM model, which is viewed as the most important cosmological model today in the cosmology community. Nevertheless, the cosmological constant is suffering from severe theoretical challenge: one cannot understand why the theoretical value of Λ from the current framework of physics is greater than the observational value by many orders of magnitude [4]. It is known that the cosmological constant is equivalent to the vacuum energy, and so its value is determined by the sum of the zero-point energy of each mode of all the quantum fields; thus, we have $\rho_\Lambda \approx k_{\max}^4/(16\pi^2)$, where k_{\max} is the imposed momentum ultraviolet (UV) cutoff. Taking the UV cutoff to be the Planck scale ($\approx 10^{19}$ GeV), where one expects quantum field theory in a classical spacetime metric to breakdown, the vacuum energy density would exceed the critical density by some 120 orders of magnitude.

Obviously, the key to the problem is gravity. One should not have calculated the value of Λ or the vacuum energy density in a context without gravity. Actually, it is conjectured that the cosmological constant problem would be solved when a full theory of quantum gravity is established. However, in the present day that we have no a quantum gravity theory,

how could we understand the cosmological constant problem from a point of view of quantum gravity? In fact, this attempt has begun—a typical example is the holographic dark energy model [5] which originates from the holographic principle [6] of quantum gravity. It is expected that the theoretical and phenomenological studies on holographic dark energy might provide significant clues for the bottom-up exploration of a full quantum theory of gravity.

When considering gravity in a quantum field system, the conventional local quantum field theory would break down due to the too many degrees of freedom that would cause the formation of a black hole. However, once the holographic principle is considered, the number of degrees of freedom could be reduced. One could put an energy bound on the vacuum energy density, $\rho_\Lambda L^3 \leq M_{\text{Pl}}^2 L$, where M_{Pl} is the reduced Planck mass, which implies that the total energy in a spatial region with size L should not exceed the mass of a black hole with the same size [7]. The largest length size compatible with this bound is the infrared (IR) cutoff size of this effective quantum field theory. Evidently, the holographic principle gives rise to a dark energy model basing on the effective quantum field theory with a UV/IR duality. From the UV/IR correspondence, the UV problem of dark energy can be converted into an IR problem. A given IR scale can saturate that bound, and so one can write the dark energy density as $\rho_\Lambda = 3c^2 M_{\text{Pl}}^2 L^{-2}$ [5], where c is a phenomenological parameter (dimensionless) characterizing all of the uncertainties of the theory. This indicates that the UV cutoff of the theory would not be fixed but run with the evolution of the IR cutoff, i.e., $k_{\max} \propto L^{-1/2}$. The original holographic dark energy model chooses the event horizon of the universe as the IR cutoff of the theory, explaining the fine-tuning problem and the coincidence problem at the same time in some degree [5]. Actually, it is clear to see that the holographic dark energy is essentially a holographic vacuum energy [8]. However, this holographic vacuum energy does not behave like a usual vacuum energy, owing to the fact that its equation of state (EOS) parameter w

*Corresponding author

†Electronic address: zhangxin@mail.neu.edu.cn

is not equal to -1 . To keep the form of the stress-energy of the vacuum, $T_{\Lambda}^{\mu\nu} = \rho_{\Lambda} g^{\mu\nu}$, required by general covariance, one possible way is to let the vacuum exchange energy with dark matter. This requires that we fix the EOS, $w = -1$, for the holographic vacuum energy, resulting in the continuity equations for Λ and CDM, $\dot{\rho}_{\Lambda} = -Q$ and $\dot{\rho}_m + 3H\rho_m = Q$, where Q describes the interaction between Λ and CDM; note that, although ρ_m includes densities of cold dark matter and baryon matter, in this place we use ρ_m to approximately describe dark matter density due to the fact that the density of baryon matter is much less than that of dark matter. We call this model the “holographic $\Lambda(t)$ CDM model.”

In fact, the possibility that Λ is not a real constant but is time variable or dynamical was considered many years ago [9]. Usually, one specifies a time-dependence form for $\Lambda(t)$ by hand and then establishes a phenomenological $\Lambda(t)$ CDM model [10]. In our holographic $\Lambda(t)$ CDM model, however, $\Lambda(t)$ originates from the holographic principle of quantum gravity, and the scenario is established based on the effective quantum field theory combining with the fundamental principle of quantum gravity. Now, as usual, the problem becomes how to choose an appropriate IR cutoff for the theory. As mentioned above, the original holographic dark energy model takes the event horizon as the IR cutoff, and this model setting obtained great successes in both theoretical and observational aspects [11, 12]. Nevertheless, a criticism concerning the causality problem arose for this model: the existence of the event horizon should be a consequence but should not be a premise of a dark energy model. This criticism gave rise to other versions of holographic dark energy, such as agegraphic dark energy model [13] and Ricci dark energy model [14]. However, a recent study on inflation and quantum mechanics [15] may remove the causality problem from the holographic dark energy model. In Ref. [15], the authors showed that, if inflation indeed happened in the early times, the quantum no-cloning theorem requires that the existence of event horizon is a must. So, once inflation is accommodated in a cosmological scenario, the existence of the event horizon could be adopted as a premise of a holographic dark energy model. On the other hand, the holographic dark energy model based on the event horizon is much better than other versions of the holographic model in fitting the observational data [16]. For the various versions of the holographic $\Lambda(t)$ CDM model (i.e., the versions taking the IR cutoff as the Hubble scale, particle horizon, event horizon, age of the universe, conformal time of the universe, etc), it has also been shown that the version concerning the event horizon fits the observational data best [17]. Thus, based on these facts, in this paper, we only consider the holographic $\Lambda(t)$ CDM model in which the event horizon of the universe provides the effective quantum field theory with the IR length-scale cutoff.

In this paper, we study the holographic $\Lambda(t)$ CDM model in a non-flat universe. It is well known that the flatness of the observable universe is one of the predictions of conventional inflation models. Though inflation theoretically produces Ω_{k0} on the order of the magnitude of quantum fluctuations, namely, $\Omega_{k0} \sim 10^{-5}$, the current observational limit on Ω_{k0} is of order 10^{-2} [3]. In addition, since the spatial curva-

ture is degenerate with the parameters of dark energy models, there is a revived and growing interest in studying dark energy models with spatial curvature. In the following, we shall first derive the evolution equations of the holographic $\Lambda(t)$ CDM model and then place the current observational constraints on the model.

The paper is organized as follows: In Sec. II we will derive the evolution equations of the holographic $\Lambda(t)$ CDM model in a non-flat universe, the cosmological constraints and the result discussions are presented in Sec. III, and the conclusion is given in Sec. IV.

II. THE HOLOGRAPHIC $\Lambda(t)$ CDM MODEL

Consider the Friedmann-Robertson-Walker universe with the metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (1)$$

where $k = 1, 0, -1$ for closed, flat, and open geometries, respectively, and $a(t)$ is the scale factor of the universe with the convention $a(t_0) = 1$. The Friedmann equation is

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{\Lambda}) - \frac{k}{a^2}, \quad (2)$$

where $H = \dot{a}/a$ is the Hubble parameter. Since we focus only on the late-time evolution of the universe, the radiation component ρ_{rad} is negligible.

By definition of the holographic vacuum energy, we have

$$\rho_{\Lambda} = 3c^2 M_{\text{Pl}}^2 L^{-2}, \quad (3)$$

where L is the IR length-scale cutoff of the theory,

$$L = ar(t), \quad (4)$$

with $r(t)$ determined by

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^{+\infty} \frac{dt}{a(t)}. \quad (5)$$

From the above equation, one can easily obtain

$$r(t) = \frac{1}{\sqrt{k}} \sin \left(\sqrt{k} \int_t^{+\infty} \frac{dt}{a} \right) = \frac{1}{\sqrt{k}} \sin \left(\sqrt{k} \int_{a(t)}^{+\infty} \frac{da}{Ha^2} \right). \quad (6)$$

The Friedmann equation (2) can be rewritten as

$$\Omega_m + \Omega_{\Lambda} + \Omega_k = 1, \quad (7)$$

where

$$\Omega_k = \frac{-k}{H^2 a^2} = \frac{\Omega_{k0} a^{-2}}{(H/H_0)^2}, \quad (8)$$

and

$$\Omega_{\Lambda} = \frac{\rho_{\Lambda}}{3M_{\text{Pl}}^2 H^2} = \frac{c^2}{H^2 L^2}. \quad (9)$$

From Eq. (9), one has $L = c/(H\sqrt{\Omega_\Lambda})$, and thus

$$r(t) = \frac{L}{a} = \frac{c}{aH\sqrt{\Omega_\Lambda}}. \quad (10)$$

Combining Eqs. (6) and (10), we have

$$\arcsin \frac{c\sqrt{k}}{aH\sqrt{\Omega_\Lambda}} = \sqrt{k} \int_t^{+\infty} \frac{dt}{a}. \quad (11)$$

Taking the derivatives with respect to time t for the both sides of the above equation, we obtain

$$\frac{\dot{\Omega}_\Lambda}{2\Omega_\Lambda H} + 1 + \frac{\dot{H}}{H^2} = \sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k}. \quad (12)$$

In our model, as mentioned above, the vacuum exchanges energy with dark matter, so the continuity equations for them are

$$\dot{\rho}_\Lambda = -Q, \quad (13)$$

$$\dot{\rho}_m + 3H\rho_m = Q. \quad (14)$$

This means that $\dot{\rho}_\Lambda + \dot{\rho}_m + 3H\rho_m = 0$ or $\dot{\rho}_{\text{cr}} - \dot{\rho}_k + 3H(\rho_{\text{cr}} - \rho_\Lambda - \rho_k) = 0$, where $\rho_{\text{cr}} = 3M_{\text{pl}}^2 H^2$ is the critical density of the universe. Using $\dot{\rho}_{\text{cr}} = 2(\dot{H}/H)\rho_{\text{cr}}$ and $\dot{\rho}_k = -2H\rho_k$, one gets

$$\frac{\dot{H}}{H^2} = \frac{1}{2}(3\Omega_\Lambda + \Omega_k - 3). \quad (15)$$

Substituting this relation into Eq. (12), one obtains the expression

$$\dot{\Omega}_\Lambda = 2H\Omega_\Lambda \left[\sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k} - \frac{1}{2}(3\Omega_\Lambda + \Omega_k - 1) \right]. \quad (16)$$

Differential equations (15) and (16) govern the cosmological evolution of the holographic $\Lambda(t)$ CDM model. In practice, we replace the variable t with the variable z , and derive the following two differential equations

$$\frac{1}{(H/H_0)} \frac{d(H/H_0)}{dz} = -\frac{1}{2(1+z)}(3\Omega_\Lambda + \Omega_k - 3), \quad (17)$$

$$\frac{d\Omega_\Lambda}{dz} = -\frac{\Omega_\Lambda}{1+z} \left(2\sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k} - 3\Omega_\Lambda - \Omega_k + 1 \right). \quad (18)$$

Solving the differential equations (17) and (18), we can learn the evolution behavior of $H(z)$ and $\Omega_\Lambda(z)$, and then various observables can also be obtained. We will numerically solve these two equations and use the solutions to fit with the observational data.

One may also be interested in the effective EOS parameters of the holographic vacuum energy and the cold dark matter. Equations (13) and (14) can be rewritten as

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\text{eff}})\rho_\Lambda = 0, \quad (19)$$

$$\dot{\rho}_m + 3H(1 + w_m^{\text{eff}})\rho_m = 0, \quad (20)$$

where $w_\Lambda^{\text{eff}} = -1 + Q/(3H\rho_\Lambda)$ and $w_m^{\text{eff}} = -Q/(3H\rho_m)$ are the effective EOS parameters for holographic vacuum energy and cold dark matter, respectively. One can easily obtain the expressions

$$w_\Lambda^{\text{eff}} = -\frac{1}{3} \left(2\sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k} + 1 \right), \quad (21)$$

$$w_m^{\text{eff}} = \frac{2\Omega_\Lambda}{3(1 - \Omega_\Lambda - \Omega_k)} \left(\sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k} - 1 \right). \quad (22)$$

The deceleration parameter $q \equiv -\ddot{a}/(aH^2)$ can also be easily derived,

$$q = -\frac{1}{2}(3\Omega_\Lambda + \Omega_k - 1). \quad (23)$$

So, one sees that, once the solutions $H(z)$ and $\Omega_\Lambda(z)$ are obtained from the differential equations (17) and (18), these quantities are known to us.

III. COSMOLOGICAL CONSTRAINTS

In this section, we will constrain the holographic $\Lambda(t)$ CDM model by using the current observational data. We will use the 557 SN data from the Union2 dataset, the CMB data from the WMAP 7-year observation, and the BAO data from the SDSS. We will obtain the best-fitted parameters and likelihoods by using a Markov Chain Monte Carlo (MCMC) method.

We use the data points of the 557 Union2 SN compiled in Ref. [18]. The theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (24)$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ with h the Hubble constant H_0 in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the Hubble-free luminosity distance

$$D_L(z) = \frac{1+z}{\sqrt{|\Omega_{k0}|}} \text{sinn} \left(\sqrt{|\Omega_{k0}|} \int_0^z \frac{dz'}{E(z')} \right), \quad (25)$$

where $E(z) \equiv H(z)/H_0$, and

$$\frac{\text{sinn}(\sqrt{|\Omega_{k0}|}x)}{\sqrt{|\Omega_{k0}|}} = \begin{cases} \sin(\sqrt{|\Omega_{k0}|}x)/\sqrt{|\Omega_{k0}|}, & \text{if } \Omega_{k0} < 0, \\ x, & \text{if } \Omega_{k0} = 0, \\ \sinh(\sqrt{|\Omega_{k0}|}x)/\sqrt{|\Omega_{k0}|}, & \text{if } \Omega_{k0} > 0. \end{cases}$$

The χ^2 function for the 557 Union2 SN data is given by

$$\chi_{\text{SN}}^2 = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}, \quad (26)$$

where σ is the corresponding 1σ error of distance modulus for each supernova. The parameter μ_0 is a nuisance parameter and one can expand Eq. (26) as

$$\chi_{\text{SN}}^2 = A - 2\mu_0 B + \mu_0^2 C, \quad (27)$$

where A , B and C are defined as in Ref. [19]. Evidently, Eq. (27) has a minimum for $\mu_0 = B/C$ at

$$\tilde{\chi}_{\text{SN}}^2 = A - \frac{B^2}{C}. \quad (28)$$

Since $\chi_{\text{SN}, \min}^2 = \tilde{\chi}_{\text{SN}, \min}^2$, instead minimizing χ_{SN}^2 we will minimize $\tilde{\chi}_{\text{SN}}^2$ which is independent of the nuisance parameter μ_0 .

Next, we consider the cosmological observational data from WMAP and SDSS. For the WMAP data, we use the CMB shift parameter R ; for the SDSS data, we use the parameter A of the BAO measurement. It is widely believed that both R and A are nearly model-independent and contain essential information of the full WMAP CMB and SDSS BAO data [20]. The shift parameter R is given by [20, 21]

$$R \equiv \frac{\sqrt{\Omega_{\text{m}0}}}{\sqrt{|\Omega_{k0}|}} \text{sinn} \left(\sqrt{|\Omega_{k0}|} \int_0^{z_*} \frac{dz'}{E(z')} \right), \quad (29)$$

where the redshift of recombination $z_* = 1091.3$, from the WMAP 7-year data [3]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in the CMB power spectrum of temperature fluctuations [20, 21]. The value of R is 1.725 ± 0.018 , from the WMAP 7-year data [3]. The distance parameter A from the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [22] is given by

$$A \equiv \frac{\sqrt{\Omega_{\text{m}0}}}{E(z_b)^{\frac{1}{3}}} \left[\frac{1}{z_b \sqrt{|\Omega_{k0}|}} \text{sinn} \left(\sqrt{|\Omega_{k0}|} \int_0^{z_b} \frac{dz'}{E(z')} \right) \right]^{\frac{2}{3}}, \quad (30)$$

where $z_b = 0.35$. In Ref. [23], the value of A has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here, the scalar spectral index n_s is taken to be 0.963, from the WMAP 7-year data [3]. So the total χ^2 is given by

$$\chi^2 = \tilde{\chi}_{\text{SN}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2, \quad (31)$$

where $\tilde{\chi}_{\text{SN}}^2$ is given by (28), $\chi_{\text{CMB}}^2 = (R - R_{\text{obs}})^2/\sigma_R^2$ and $\chi_{\text{BAO}}^2 = (A - A_{\text{obs}})^2/\sigma_A^2$. The best-fitted model parameters are determined by minimizing the total χ^2 .

Now, we fit the holographic $\Lambda(t)$ CDM model to the observational data. We use the MCMC method and finally we obtain the best-fit values and 1σ , 2σ , and 3σ values for the model parameters. Since the Λ CDM model is an important reference model for the studies of dark energy models, we also fit the standard Λ CDM model to the same data for comparison. The models are studied in the cases of flat and non-flat universes, respectively. So, the calculations are performed for four cases—flat Λ CDM, non-flat Λ CDM, flat $\Lambda(t)$ CDM, and non-flat $\Lambda(t)$ CDM. The best-fit and 1σ values of the parameters with χ_{\min}^2 of the four models are all presented in Table I.

For the flat holographic $\Lambda(t)$ CDM model, we obtain the best-fit values of the parameters: $c = 0.694$ and $\Omega_{\text{m}0} = 0.256$, and the corresponding minimal χ^2 is $\chi_{\min}^2 = 545.080$. Obviously, the χ_{\min}^2 value of this model is much greater than that of the flat Λ CDM model, even though in the case of number of parameters being greater by one. The fit value of $\Omega_{\text{m}0}$ is also

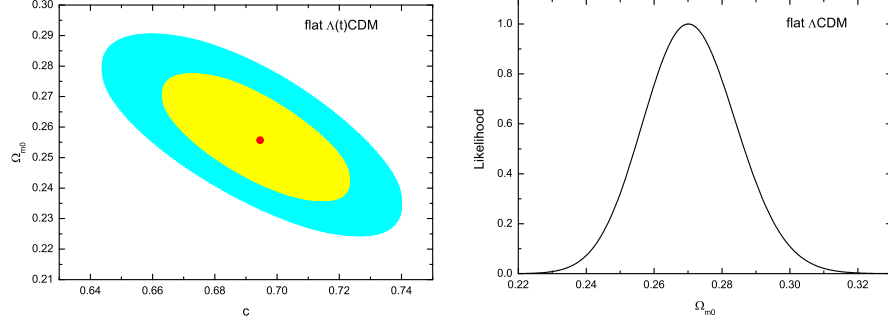
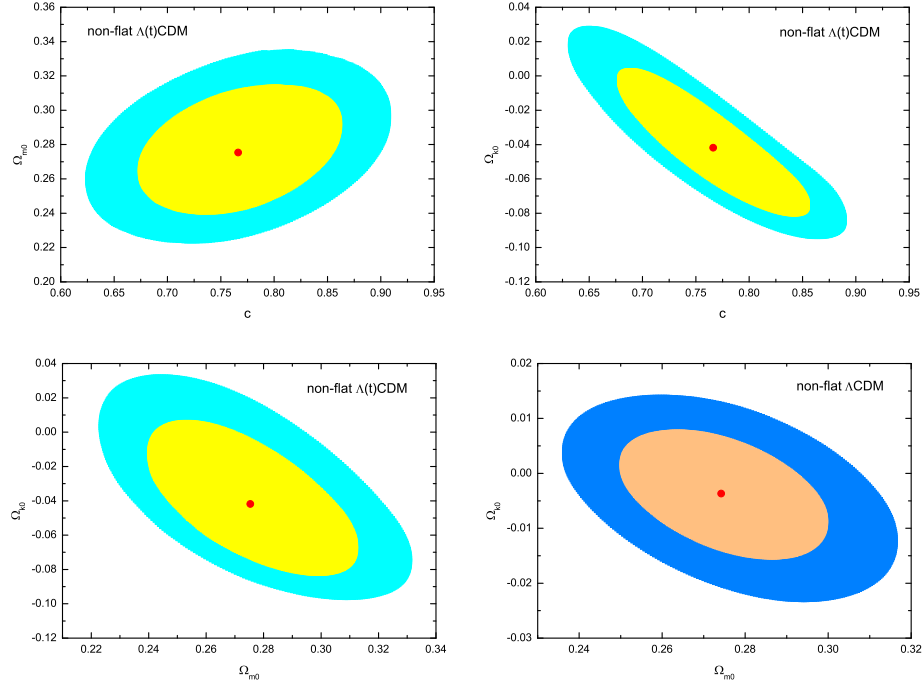
a little bit small, 0.256, evidently less than that of the Λ CDM model, 0.270. Now, let us see the fit results of the holographic $\Lambda(t)$ CDM model in the case of a non-flat universe. For this case, we have $c = 0.766$, $\Omega_{\text{m}0} = 0.275$, $\Omega_{k0} = -0.042$, and $\chi_{\min}^2 = 542.803$. Comparing to the non-flat Λ CDM model, we find that the χ_{\min}^2 values are similar (for the Λ CDM model, $\chi_{\min}^2 = 542.699$), and the $\Omega_{\text{m}0}$ best-fit values are also very similar (for the Λ CDM model, $\Omega_{\text{m}0} = 0.274$). Comparing the holographic $\Lambda(t)$ CDM model in the flat and non-flat universes, we find that the spatial curvature plays an important role in the model in fitting the data. Once adding the parameter Ω_{k0} in the model, the χ_{\min}^2 value decreases by a very distinct amount, and correspondingly, of course, the ranges of the parameters amplify to some extent as usual. We find that the holographic $\Lambda(t)$ CDM model tends to favor a spatially closed universe (the best-fit value of Ω_{k0} is -0.042), and we find that the 95% confidence level (CL) range for the spatial curvature is $-0.101 < \Omega_{k0} < 0.040$.

Figures 1 and 2 show the probability contours at 68% and 95% CLs in the parameter planes for the holographic $\Lambda(t)$ CDM model and the Λ CDM model in flat and non-flat universes. In Fig. 1, we show the constraint results for the models in a flat universe. In the left panel we plot the contours in the c – $\Omega_{\text{m}0}$ plane for the $\Lambda(t)$ CDM model, and in the right panel we plot the one-dimensional likelihood of the parameter $\Omega_{\text{m}0}$ for the Λ CDM model. We find that, for the flat $\Lambda(t)$ CDM model, the parameters c and $\Omega_{\text{m}0}$ are anti-correlated. In Fig. 2, we show the constraint results for the models in a non-flat universe. In this figure, the contours in the c – $\Omega_{\text{m}0}$, c – Ω_{k0} and $\Omega_{\text{m}0}$ – Ω_{k0} planes for the non-flat $\Lambda(t)$ CDM model are shown in the upper-left, upper-right and lower-left panels, respectively, and the contours in the $\Omega_{\text{m}0}$ – Ω_{k0} plane for the non-flat Λ CDM model is shown in the lower-right panel. Interestingly, we find that, since c and Ω_{k0} are anti-correlated, and $\Omega_{\text{m}0}$ and Ω_{k0} are also anti-correlated, c and $\Omega_{\text{m}0}$ in the $\Lambda(t)$ CDM model have a positive correlation in this case.

Next, we shall analyze the cosmological evolution of the holographic $\Lambda(t)$ CDM model using the fit results. In the standard Λ CDM model, Λ is a real constant, and so the vacuum energy density ρ_{Λ} remains constant during the expansion of the universe. However, in the holographic $\Lambda(t)$ CDM model, though the EOS of the vacuum remains -1 , the cold dark matter component exchanges energy with the vacuum, so that the vacuum energy density ρ_{Λ} is not a constant but dynamically evolves during the expansion of the universe. Thus, we want to see how ρ_{Λ} evolves in the $\Lambda(t)$ CDM model. We want to learn the direction of the energy flow between $\Lambda(t)$ and CDM. For this purpose, we plot $\rho_{\Lambda}/\rho_{\text{cr}0}$ versus z in Fig. 3 by using the best-fit results, where $\rho_{\text{cr}0} = 3M_{\text{Pl}}^2 H_0^2$ is the present-day critical density of the universe. In general, we have $\rho_{\Lambda}/\rho_{\text{cr}0} = E^2 \Omega_{\Lambda}$; for the Λ CDM model this quantity is just $\Omega_{\Lambda 0}$, and for the holographic $\Lambda(t)$ CDM model this quantity can be directly obtained by using the solutions of the differential equations (17) and (18). The cases of the flat and non-flat universes are shown in the left and right panels of Fig. 3, respectively, where the solid lines denote the $\Lambda(t)$ CDM model and the horizontal dash-dot lines correspond to the Λ CDM model. From this figure, we see that, in the $\Lambda(t)$ CDM model,

TABLE I: The fit results of the Λ CDM and holographic $\Lambda(t)$ CDM models.

Model	c	Ω_{m0}	Ω_{k0}	χ^2_{\min}
flat Λ CDM	N/A	$0.270^{+0.014}_{-0.013}$	N/A	542.919
non-flat Λ CDM	N/A	$0.274^{+0.025}_{-0.024}$	$-0.004^{+0.012}_{-0.012}$	542.699
flat $\Lambda(t)$ CDM	$0.694^{+0.030}_{-0.032}$	$0.256^{+0.023}_{-0.022}$	N/A	545.080
non-flat $\Lambda(t)$ CDM	$0.766^{+0.103}_{-0.097}$	$0.275^{+0.039}_{-0.036}$	$-0.042^{+0.053}_{-0.044}$	542.803

FIG. 1: The observational constraints on the holographic $\Lambda(t)$ CDM model and the Λ CDM model in a flat universe. Left: the probability contours at 68% and 95% CLs in the c - Ω_{m0} plane for the holographic $\Lambda(t)$ CDM model. Right: The one-dimensional likelihood distribution of the parameter Ω_{m0} for the Λ CDM model.FIG. 2: The observational constraints on the holographic $\Lambda(t)$ CDM model and the Λ CDM model in a non-flat universe. Upper-left, upper-right and lower-left: the probability contours at 68% and 95% CLs in the c - Ω_{m0} , c - Ω_{k0} and Ω_{m0} - Ω_{k0} planes for the holographic $\Lambda(t)$ CDM model. Lower-right: the probability contours at 68% and 95% CLs in the Ω_{m0} - Ω_{k0} plane for the Λ CDM model.

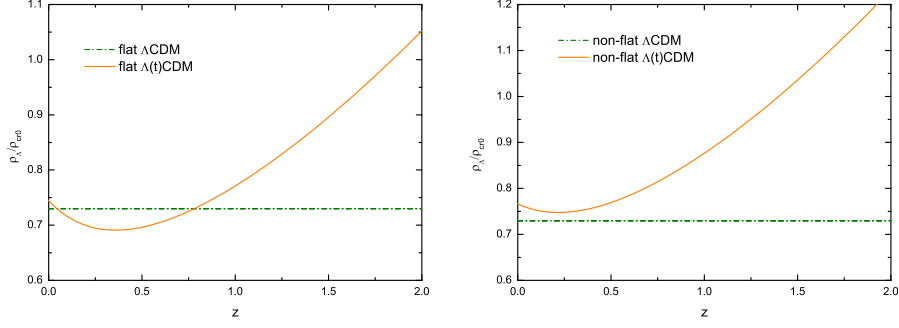


FIG. 3: The evolutions of $\rho_\Lambda/\rho_{\text{cr0}}$ in the holographic $\Lambda(t)$ CDM and Λ CDM models, in the best-fit cases.

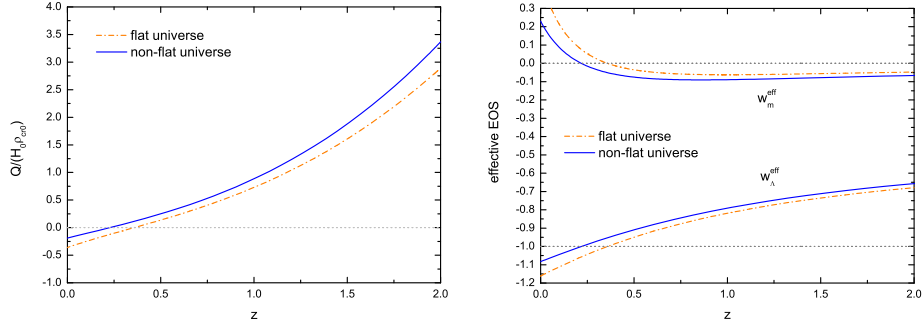


FIG. 4: The evolutions of $Q/(H_0\rho_{\text{cr0}})$ and the effective EOS parameters, w_m^{eff} and w_Λ^{eff} , in the holographic $\Lambda(t)$ CDM model, in the best-fit cases.

ρ_Λ is not a constant, but is dynamical, first decreases and then increases during the expansion of the universe, exhibiting a quintom feature. This implies that the interaction between $\Lambda(t)$ and CDM induces an energy flow of which the direction is first from $\Lambda(t)$ to CDM and then from CDM to $\Lambda(t)$. To see the direction of the energy flow more clearly, let us derive the explicit expression of Q . Combining Eqs. (13), (15), and (16), we obtain

$$\frac{Q}{H_0\rho_{\text{cr0}}} = -2E^3\Omega_\Lambda \left(\sqrt{\frac{\Omega_\Lambda}{c^2} + \Omega_k} - 1 \right). \quad (32)$$

Then, we can directly plot $Q(z)$. Figure 4 shows the evolutions of $Q/(H_0\rho_{\text{cr0}})$ and the effective EOS parameters, w_m^{eff} and w_Λ^{eff} , in the $\Lambda(t)$ CDM model, where the dash-dot lines denote the flat universe and the solid lines correspond to the non-flat universe. We see clearly that Q crosses the noninteracting line ($Q = 0$) from $Q > 0$ to $Q < 0$. This indeed indicates that at first $\Lambda(t)$ decays to CDM, and then CDM decays to $\Lambda(t)$. We can also see from the right panel of Fig. 4 that w_m^{eff} crosses the $w = 0$ line from $w < 0$ to $w > 0$, and w_Λ^{eff} crosses the $w = -1$ line from $w > -1$ to $w < -1$, at around $z = 0.2-0.3$, conforming the sign-change of Q discussed above. In fact, in Ref. [24], it has been shown that the interaction between dark energy and dark matter may change sign during the cosmological evolution. By parameterizing the coupling and fitting to the current data, it is shown in Ref. [24] that a time-varying vacuum sce-

nario is favored, in which the interaction $Q(z)$ crosses the non-interacting line during the cosmological evolution and the sign changes from negative to positive. The holographic $\Lambda(t)$ CDM model is just a time-varying vacuum energy model in which Q changes sign, however, in this model the sign changes from positive to negative, opposite to the case in Ref. [24]. This implies that we should pay more attention to the time-varying vacuum model and seriously consider the theoretical construction of a sign-changeable or oscillatory interaction between dark sectors.

IV. CONCLUSION

In this paper, we have studied the holographic $\Lambda(t)$ CDM model in a non-flat universe. We demonstrated that, in the holographic model, it is best to choose the event horizon size of the universe as the IR cutoff of the effective quantum field theory. We then derived the evolution equations of the model in a non-flat universe. It is shown that the cosmological evolution of the model is governed by the differential equations (17) and (18); solving these two differential equations, one can obtain the evolutions of $E(z)$ and $\Omega_\Lambda(z)$ directly, and other cosmological quantities can subsequently be derived. Actually, the holographic $\Lambda(t)$ CDM model is an interacting holographic vacuum energy scenario in which the interaction between $\Lambda(t)$

and CDM is described by $Q = -\dot{\rho}_\Lambda$. The explicit expression of Q is given by Eq. (32).

In order to learn the cosmological evolution described by the holographic $\Lambda(t)$ CDM model, we place observational constraints on the model, and analyze the cosmological evolution of the model using the fit results. We constrained the holographic $\Lambda(t)$ CDM model and the Λ CDM model by using the current observational data, including the 557 Union2 SN data, the CMB WMAP 7-yr data, and the BAO SDSS data. Our fit results show that the spatial curvature plays a significant role in the model in fitting the data. Once the additional parameter Ω_{k0} is involved in the model, the χ^2_{\min} value decreases by a rather distinct amount, and correspondingly, the ranges of the parameters amplify to some extent as usual. We found that the holographic $\Lambda(t)$ CDM model tends to favor a spatially closed universe (the best-fit value of Ω_{k0} is -0.042), and we found that the 95% CL range for the spatial curvature is $-0.101 < \Omega_{k0} < 0.040$. Moreover, we found that, for the holographic $\Lambda(t)$ CDM model in a flat universe, the parameters c and Ω_{m0} are anti-correlated, however, when the model is placed in a non-flat universe, c and Ω_{m0} get a positive correlation, in our data analysis.

We illustrated the cosmological evolution of the model by

taking the best-fit values of the parameters as an example. We showed that the vacuum energy density ρ_Λ is indeed not a constant in the $\Lambda(t)$ CDM model; it first decreases and then increases during the expansion of the universe, exhibiting a quintom feature. This implies that the interaction between $\Lambda(t)$ and CDM induces an energy flow of which the direction is first from $\Lambda(t)$ to CDM and then from CDM to $\Lambda(t)$. We also plotted the evolution of Q and showed that Q crosses the noninteracting line ($Q = 0$) from $Q > 0$ to $Q < 0$, at around $z = 0.2 - 0.3$. So, the holographic $\Lambda(t)$ CDM model is just a time-varying vacuum energy scenario in which Q changes sign.

Acknowledgments

This work was supported by the National Science Foundation of China under Grant Nos. 10705041, 10975032, 11047112 and 11175042, and by the National Ministry of Education of China under Grant Nos. NCET-09-0276, N100505001, N090305003, and N110405011.

-
- [1] P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75** (2003) 559; E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15** (2006) 1753; J. Frieman, M. Turner and D. Huterer, *Ann. Rev. Astron. Astrophys.* **46**, 385 (2008); M. Li, X. D. Li, S. Wang and Y. Wang, *Commun. Theor. Phys.* **56** (2011) 525; K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, *arXiv:1205.3421*.
 - [2] A. Einstein, *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1917** (1917) 142.
 - [3] E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **192** (2011) 18.
 - [4] S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1; V. Sahni and A. A. Starobinsky, *Int. J. Mod. Phys. D* **9** (2000) 373; S. M. Carroll, *Living Rev. Rel.* **4** (2001) 1; T. Padmanabhan, *Phys. Rept.* **380** (2003) 235.
 - [5] M. Li, *Phys. Lett. B* **603** (2004) 1.
 - [6] G. 't Hooft, *arXiv:gr-qc/9310026*; L. Susskind, *J. Math. Phys.* **36** (1995) 6377.
 - [7] A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Phys. Rev. Lett.* **82** (1999) 4971.
 - [8] X. Zhang, *Phys. Lett. B* **648**, 1 (2007). X. Zhang, *Phys. Rev. D* **74**, 103505 (2006); J. Zhang, X. Zhang and H. Liu, *Phys. Lett. B* **651**, 84 (2007); Y. Z. Ma and X. Zhang, *Phys. Lett. B* **661**, 239 (2008).
 - [9] M. Ozer and M. O. Taha, *Phys. Lett. B* **171**, 363 (1986); M. Ozer and M. O. Taha, *Nucl. Phys. B* **287**, 776 (1987); O. Bertolami, *Nuovo Cim. B* **93**, 36 (1986); K. Freese, F. C. Adams, J. A. Frieman and E. Mottola, *Nucl. Phys. B* **287**, 797 (1987); P. J. E. Peebles and B. Ratra, *Astrophys. J.* **325**, L17 (1988); M. S. Berman, *Phys. Rev. D* **43**, 1075 (1991).
 - [10] P. Wang and X. H. Meng, *Class. Quant. Grav.* **22**, 283 (2005); H. A. Borges and S. Carneiro, *Gen. Rel. Grav.* **37**, 1385 (2005); S. Carneiro, M. A. Dantas, C. Pigozzo and J. S. Alcaniz, *Phys. Rev. D* **77**, 083504 (2008); S. Basilakos, M. Plionis and J. Sola, *Phys. Rev. D* **80**, 083511 (2009).
 - [11] Q. G. Huang and M. Li, *JCAP* **0408**, 013 (2004); Q. G. Huang and M. Li, *JCAP* **0503**, 001 (2005); X. Zhang, *Int. J. Mod. Phys. D* **14**, 1597 (2005); J. Zhang, X. Zhang and H. Liu, *Eur. Phys. J. C* **52**, 693 (2007); Y. G. Gong, *Phys. Rev. D* **70**, 064029 (2004); B. Wang, E. Abdalla and R. K. Su, *Phys. Lett. B* **611**, 21 (2005); X. Wu, R. G. Cai and Z. H. Zhu, *Phys. Rev. D* **77**, 043502 (2008); B. Chen, M. Li and Y. Wang, *Nucl. Phys. B* **774**, 256 (2007); J. Zhang, H. Liu and X. Zhang, *Phys. Lett. B* **659**, 26 (2008); M. Li, C. Lin and Y. Wang, *JCAP* **0805**, 023 (2008); M. Li, X. D. Li, C. Lin and Y. Wang, *Commun. Theor. Phys.* **51**, 181 (2009); S. Nojiri and S. D. Odintsov, *Gen. Rel. Grav.* **38**, 1285 (2006); X. Zhang, *Phys. Lett. B* **683** (2010) 81.
 - [12] X. Zhang and F. Q. Wu, *Phys. Rev. D* **72**, 043524 (2005); X. Zhang and F. Q. Wu, *Phys. Rev. D* **76**, 023502 (2007); Q. G. Huang and Y. G. Gong, *JCAP* **0408**, 006 (2004); Z. Chang, F. Q. Wu and X. Zhang, *Phys. Lett. B* **633**, 14 (2006); J. Y. Shen, B. Wang, E. Abdalla and R. K. Su, *Phys. Lett. B* **609**, 200 (2005); Z. L. Yi and T. J. Zhang, *Mod. Phys. Lett. A* **22**, 41 (2007); Y. Z. Ma, Y. Gong and X. Chen, *Eur. Phys. J. C* **60**, 303 (2009); M. Li, X. D. Li, S. Wang, Y. Wang and X. Zhang, *JCAP* **0912** (2009) 014; Z. Zhang, S. Li, X. D. Li, X. Zhang and M. Li, *arXiv:1204.6135 [astro-ph.CO]*.
 - [13] R. G. Cai, *Phys. Lett. B* **657**, 228 (2007); H. Wei and R. G. Cai, *Phys. Lett. B* **660**, 113 (2008); J. Zhang, X. Zhang and H. Liu, *Eur. Phys. J. C* **54**, 303 (2008); X. L. Liu, J. Zhang and X. Zhang, *Phys. Lett. B* **689** (2010) 139; Y. H. Li, J. F. Zhang and X. Zhang, *arXiv:1201.5446 [gr-qc]*.
 - [14] C. Gao, F. Q. Wu, X. Chen and Y. G. Shen, *Phys. Rev. D* **79**, 043511 (2009); X. Zhang, *Phys. Rev. D* **79** (2009) 103509; C. J. Feng and X. Zhang, *Phys. Lett. B* **680** (2009) 399; R. G. Cai, B. Hu and Y. Zhang, *Commun. Theor. Phys.* **51**, 954 (2009); J. Zhang, L. Zhang and X. Zhang, *Phys. Lett. B* **691** (2010) 11.

- [15] Q. G. Huang and F. L. Lin, arXiv:1201.2443 [hep-th].
- [16] M. Li, X. D. Li, S. Wang and X. Zhang, JCAP **0906** (2009) 036; M. Li, X. D. Li and X. Zhang, Sci. China Phys. Mech. Astron. **53**, 1631 (2010).
- [17] Y. Chen, Z. H. Zhu, L. Xu and J. S. Alcaniz, Phys. Lett. B **698**, 175 (2011); Z. X. Zhai, T. J. Zhang and W. B. Liu, JCAP **1108**, 019 (2011).
- [18] R. Amanullah *et al.*, Astrophys. J. **716**, 712 (2010).
- [19] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **72**, 123519 (2005); L. Perivolaropoulos, Phys. Rev. D **71**, 063503 (2005); S. Nesseris and L. Perivolaropoulos, JCAP **0702**, 025 (2007).
- [20] Y. Wang and P. Mukherjee, Astrophys. J. **650**, 1 (2006).
- [21] J. R. Bond, G. Efstathiou and M. Tegmark, Mon. Not. Roy. Astron. Soc. **291**, L33 (1997).
- [22] M. Tegmark *et al.* [SDSS Collaboration], Astrophys. J. **606**, 702 (2004); M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **74**, 123507 (2006).
- [23] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633**, 560 (2005).
- [24] Y. H. Li and X. Zhang, Eur. Phys. J. C **71**, 1700 (2011).